x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	–4	4	2
4	–1	3	6	7

continuous

The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x. The function h is given by $h(x) = f(g(x)) - 6$.

• prove about function I have? IVT

use a theorem • prove about derivative of function?

(a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.

Since f and g are differentiable, they are continuous, and $-7 < -5 < 3$, there is a value r, $1 < r < 3$, such that $h(r) = -5$.

$h(1) = f(g(1)) - 6$
$f(2) - 6$
$9 - 6 = 3$
$h(3) = f(g(3)) - 6$
$f(4) - 6$
$-1 - 6 = -7$

MVT

(b) Explain why there must be a value c for $1 < c < 3$ such that $h'(c) = -5$.

Since f and g are differentiable, they are continuous, there is a value c, $1 < c < 3$, such that $h'(c) = \frac{h(3) - h(1)}{3 - 1} = \frac{-7-3}{2} = -\frac{10}{2} = -5$.

$g(1) = 2$

(d) If g^{-1} is the inverse function of g, write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.

pt. $g^{-1}(2) = 1$ (b/c $g(1) = 2$)

slope: $(g^{-1})'(2) = \frac{1}{g'(g^{-1}(2))} = \frac{1}{g'(1)} = \frac{1}{5}$

$$\boxed{y - 1 = \frac{1}{5}(x - 2)}$$

Multiple Choice and FRQ practice

1. If f is continuous for $a \le x \le b$ and differentiable for $a < x < b$, which of the following could be false?

IVT, EVT

MVT

(A) $f'(c) = \frac{f(b)-f(a)}{b-a}$ for some c such that $a < c < b$ MVT

(B) $f'(c) = 0$ for some c such that $a < c < b$

(C) f has a minimum value on $a \le x \le b$ EVT

(D) f has a maximum value on $a \le x \le b$ EVT

(E) $\int_a^b f(x)\,dx$ exists.

2. The function f is continuous for $-2 \le x \le 1$ and differentiable for $-2 < x < 1$. If $f(-2) = -5$ and $f(1) = 4$, which of the following statements could be false?

IVT, EVT

MVT

(A) There exists c, where $-2 < c < 1$, such that $f(c) = 0$ IVT bc $-5<0<4$

(B) There exists c, where $-2 < c < 1$, such that $f'(c) = 0$

(C) There exists c, where $-2 < c < 1$, such that $f(c) = 3$ IVT bc $-5<3<4$

(D) There exists c, where $-2 < c < 1$, such that $f'(c) = 3$

$\frac{f(1)-f(-2)}{1+2}$ $\frac{4+5}{3}$ $\frac{9}{3} = 3$

3. The function f is defined by $f(x) = 4x^2 - 5x + 1$. The application of the Mean Value Theorem to f on the interval $0 < x < 2$ guarantees the existence of a value c, where $0 < c < 2$ such that $f'(c) =$

$f'(c) =$

(A) 1 (B) 3 (C) 7 (D) 8

$f'(c) = \frac{f(2)-f(0)}{2-0} = \frac{7-1}{2} = \frac{6}{2} = 3$

$f'(c) = 3$

---

4. Let f be a function with first derivative defined by $f'(x) = \frac{2x^2-5}{x^2}$ for $x > 0$. It is known that $f(1) = 7$ and $f(5) = 11$. What value of x in the open interval (1, 5) satisfies the conclusion of the Mean Value Theorem for f on the closed interval [1, 5]?

(A) 1 (B) $\sqrt{\frac{5}{2}}$ (C) $\sqrt[3]{10}$ (D) $\sqrt{5}$

$f'(c) = \frac{f(5)-f(1)}{5-1}$

$= \frac{11-7}{4}$ $\frac{4}{4} = 1$

$f'(c) = 1$

$f'(x) = \frac{2x^2-5}{x^2} = 1$

$2x^2 - 5 = x^2$

$x^2 = 5$

$\boxed{x = \sqrt{5}}$

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

continuous

$c'(t)=2$

Is there a time t, $2 \le t \le 4$ at which $C'(2) = 2$? Justify your answer.

MVT

$\frac{C(4)-C(2)}{4-2}$ $\frac{12.8-8.8}{2}$ $\frac{4}{2}=2$

Since C is differentiable, it is continuous, so there is a time, t, $2<t<4$, such that $C'(t)=2$.

Ex. (2018 AB/BC 4)

t (years)	2	3	5	7	10
$H(t)$ (meters)	1.5	2	6	11	15

H & H' diff → both cont.

The height of a tree at time t is given by a twice-differentiable function H, where $H(t)$ is measured in meters and t is measured in years. Selected values of $H(t)$ are given in the table above.

(a) Use the data in the table to estimate $H'(6)$. Using correct units, interpret the meaning of $H'(6)$ in the context of the problem.

$H'(6) \approx \frac{H(7)-H(5)}{7-5} = \frac{11-6}{2}$ $\frac{5}{2}$ $\frac{\text{meters}}{\text{year}}$

NUT

when $t=6$ years, the height of the tree is increasing at a rate of $\frac{5}{2}$ meters per year.

(b) Explain why there must be at least one time t, for $2 < t < 10$, such that $H'(t) = 2$.

$\frac{15-1.5}{8} = \frac{13.5}{8} \neq 2$ go to smaller intervals inside $[2,10]$ ⟹ $\frac{h(5)-h(3)}{5-3}$

since H is diff on $[3,5]$, it is continuous on $[3,5]$, so there is a time, t, $3<t<5$, $H'(t) = \frac{H(5)-H(3)}{5-3} = \frac{6-2}{2} = \frac{4}{2} = 2$.

HW #21 -- Worksheet on Mean Value Theorem